Manufacturer's Remanufacturing Mode Selection Strategy Based on Consumer Preference

Yue Zhang, Rong Zhang
Institute of Logistics Science and Engineering, Shanghai Maritime University, Shanghai 201306, China.

Abstract
In order to find the most suitable supply chain and the most beneficial remanufacturing mode for original manufacturers (OEMs), this paper constructs a closed-loop supply chain model under the direct leadership of three OEMs by applying the game theory based on the heterogeneous demand market of consumers. By comparing and analyzing the three remanufacturing modes, it can be concluded that OEM is the best to adopt the manufacturer outsourcing remanufacturing mode. Then, the remanufacturing model of OEM introducing retailers is constructed, and the results are as follows: (1) OEM can transfer remanufacturing revenue to increase revenue through outsourcing remanufacturing or authorized remanufacturing mode; (2) OEM is best to adopt outsourcing remanufacturing mode, followed by authorized remanufacturing mode, and the most unfavorable is to adopt independent remanufacturing mode; (3) Under certain conditions, compared with the manufacturer's outsourcing remanufacturing mode, the retailer's outsourcing remanufacturing mode after OEM introduced retailers is better.

Keywords
Closed-loop Supply Chain; Remanufacturing; Outsourcing; Authorization; Independent Remanufacturing

1. Introduction
With the rapid development of social economy and technology, the problem of waste products has become increasingly prominent. In real life, for the reasons of cost recovery, brand building or lack of remanufacturing technology and ability, some manufacturers recycle and remanufacture waste products, which provides development opportunities for third-party remanufacturing. Many third-party companies recycle waste products for remanufacturing and sales without the authorization or commission of the manufacturer, and manufacturers will take measures to influence the behavior of remanufacturers in order to protect their own interests. For example, Apple's remanufacturing business in Europe and the United States has adopted an outsourcing model. Consumers can purchase Apple's remanufactured products from the official website or physical stores. In 2015, they launched the trade-in activities for iPhone and iPad products in mainland China, and authorized Foxconn to remanufacture and resell its used products [1]. The construction machinery production company Caterpillar has authorized Nanjing Lishenghong Machinery Co., Ltd. to be responsible for the subcontracting and resale business of the remanufacturing of Caterpillar machinery products in Jiangsu Province [2].

In recent years, remanufacturing has attracted the attention of scholars at home and abroad. Research on Outsourcing Remanufacturing Mode: Yan [3] et al. developed two outsourcing models to study the manufacturer should to outsource the remanufacturing business to a third-party remanufacturer or to a supplier; Zou [4] et al. used a three-stage game model to analyze the impact on other members of the supply chain when manufacturers outsourced
remanufacturing business; Zheng [5] et al. conducted a comparative analysis on two outsourcing models that directly outsource manufacturers to third-party remanufacturers and retailers subcontract to third-party remanufacturers; Ferrer and Whybark [6] studied and analyzed the manufacturer’s decision to remanufacture itself or outsource the remanufacturing business to a third-party remanufacturer. Research on authorized remanufacturing model: Ma [7] et al. researched that in the case of new products and remanufactured competition, the authorized remanufacturing model is the best choice for manufacturers; Huang [8] et al. conducted a comparative analysis of the three remanufacturing modes of manufacturer’s own remanufacturing, manufacturer-authorized distributor remanufacturing, and manufacturer-authorized third-party recycler remanufacturing; Xiong [9] et al. studied the third-party remanufacturing closed-loop supply chain model in the context of the original manufacturer’s patent protection, but the article did not consider consumers’ purchasing preferences for new products and remanufactured products; Huang [10] et al. compared the two models of retailer remanufacturing and third-party remanufacturing under patent protection, and the study found that manufacturers can maximize profits when the third party recycles and remanufactures the manufacturers. Hong [11] et al. studied the fixed authorized remanufacturing and unit authorized remanufacturing models in the Cournot duopoly model; Yang [12] et al. studied the recycling and remanufacturing model of waste products after the third-party remanufacturer paid the authorization fee to the original equipment manufacturer, and analyzed the influence of the unit authorization fee and other parameters on the decision variables and optimal profit; Zhao [13] et al. studied the three models of manufacturer’s remanufacturing, manufacturer’s unit authorized retailer remanufacturing model, and manufacturer’s fixed authorized retailer remanufacturing. Through comparative analysis that the retailer’s fixed patent authorized remanufacturing model has reduced The price of new products and remanufactured products can achieve maximum profit. Research on the independent remanufacturing model: Wu [14] and Bulmus [15] constructed a game model between original manufacturer and remanufacturer to study the influence of independent remanufacturing on manufacturing and remanufacturing activities under sales and recycling. Obtaining the original manufacturer to protect the market share of the new product will reduce the market supply of the new product.

The above documents provide new ideas for the selection of remanufacturing model strategies, but there are still some shortcomings: 1. Most of the above studies only consider a single outsourcing remanufacturing model [3-6] or authorized remanufacturing model [7-13] There are few documents comparing the advantages and disadvantages of closed-loop supply chains under different remanufacturing models, and studying how manufacturers should choose remanufacturing models to maximize their own interests. For example: Zou [1] analyzed two modes of outsourcing remanufacturing and authorized remanufacturing, and the research showed that when consumers have higher valuations of remanufactured products, third parties prefer the outsourcing mode, while manufacturers are more willing to choose outsourcing remanufacturing. Zhang [16] et al. studied a competitive closed-loop supply chain with two original equipment manufacturers and two third-party remanufacturers. They considered the two third-party remanufacturing modes of outsourcing and authorization, OEMs with duopoly in research prefer outsourcing strategies; Xia [17] et al. considered brand manufacturers to outsource the production of new products to OEMs and analyzed consumers’ best decisions for brand manufacturers and OEMs under different remanufacturing models. 2. In real life, in addition to third-party remanufacturers, original manufacturers have the flexibility to outsource remanufacturing to retailers. The existing remanufacturing theories ignore this flexibility and usually only emphasize the interaction between the original manufacturer and the third party. However, retailers and original manufacturers have mature upstream/downstream cooperation and mature sales channels. Compared with third-party
remanufacturing, retailers can better control the substitutability of products, thereby affecting the optimal pricing strategy of the closed-loop supply chain.

Therefore, the research conducted in this article on the basis of the existing ones mainly focuses on: 1. Construct four remanufacturing game models based on consumers' different preferences for new products and remanufactured products, and analyze and compare the impact of different remanufacturing models on market competition; 2. Based on the three remanufacturing models of the OEM-TPR-consumer closed-loop supply chain directly led by the OEM, analyze the remanufacturing model with the largest OEM profit, and further consider whether the OEM will introduce retailers into remanufacturing activities and analyze the situation The introduction of retailers is most beneficial to the OEM, and it provides a theoretical reference for the choice of OEM's remanufacturing strategy.

Fig. 1 Remanufacturing model

2. Problem description and model assumptions

2.1. Problem description

Consider an OEM that does not have remanufacturing capabilities, and discuss the OEM's remanufacturing business selection strategy. (1) The manufacturer's outsourcing remanufacturing model (model O). OEM directly outsourcing the remanufacturing business to TPR and pays the outsourcing fee. TPR is responsible for the recycling and remanufacturing of waste products, and OEM is responsible for the sales of new products and remanufactured products. (2) The unit authorized remanufacturing model (model U), the OEM directly authorizes the remanufacturing business to TPR and charges a unit authorization fee for each unit of remanufactured by TPR. At this time, TPR is responsible for the recycling and
remanufacturing of waste products and remanufacturing. OEM is responsible for the production and sales of new products. (3) Independent remanufacturing mode (model D), OEM does not participate in remanufacturing activities, TPR is responsible for the recycling, remanufacturing and sales of waste products, and OEM is responsible for the production and sales of new products. (4) Retailer outsourcing remanufacturing model (model S), OEM is the first-level leader, retailer is the second-level leader, and TPR is the follower. In the model, the OEM is responsible for producing new products and authorizing the remanufacturing business to the retailer and charging the authorization fee. Then the retailer outsources the remanufacturing business to TPR and pays the outsourcing fee. The retailer is responsible for the sales of new products and remanufactured products. The four remanufacturing model diagrams are shown in Fig 1:

Based on the Stackelberg game, this paper discusses the CLSC decision-making in the above four modes, and compares the optimal decision and optimal profit.

2.2. Model assumptions

This article uses $\Pi^j_i$ to represent the profit of supply chain member $j$ under model $i$, where $i \in \{O, U, D, S\}$ represents the manufacturer’s outsourced remanufacturing model, unit authorized remanufacturing model, independent remanufacturing model and retailer respectively. Outsourcing remanufacturing model, $j \in \{M, R, T\}$ represents OEM, retailer and TPR respectively. Set $c_n, c_r, T_r$ represent the production cost of new products, the production cost of remanufactured products and the outsourcing unit price of remanufactured products commissioned by TPR by manufacturers or retailers, among which $c_n > c_r, T_r > c_r$; let $f$ denote the unit authorization fee; $q_n, q_r$ represent the quantity of new products and reproductions; $p_n, p_r$ represent the sales price of new products and reproductions, where $p_n > p_r$.

Assuming that the total market size is 1, considering the heterogeneity of consumers, consumers' willingness to pay for new products is assumed $\theta$ obey uniform distribution, that is $\theta \sim U[0,1]$; Consumers' preference for new products is 1, and their preference for reproductions is $\alpha, 0 < \alpha < 1$. The utility of consumers to buy new products is $u_n = \theta - p_n$, the utility of purchasing reproductions is $u_r = \alpha \theta - p_r$. Assuming that the total market size planning is 1, considering the heterogeneity of consumer preferences, consumers' willingness to pay for new products is assumed $\theta$ obey uniform distribution, that is $\theta \sim U[0,1]$; Consumers' preference for new products is 1, and their preference for reproductions is $\alpha, 0 < \alpha < 1$. Therefore, the utility of consumers to buy new products is as follows $u_n = \theta - p_n$, the utility of purchasing reproductions is $u_r = \alpha \theta - p_r$. Therefore, the conditions for consumers to buy new products are $u_n > 0$ and $u_n > u_r$. The conditions for purchasing reproductions are $u_r > 0$ and $u_r > u_n$.

Easy to know the needs of new products and reproductions are $q_n = 1 - \frac{p_n - p_r}{1 - \alpha}$; $q_r = \frac{\alpha p_n - p_r}{\alpha (1 - \alpha)}$.

Suppose the cost of recovery effort $C = \frac{1}{2} \eta q_r^2$, where $\eta > 0$ is the scale parameter, which reflects the difficulty of recovery.

It is assumed that all recycled products can be remanufactured and sold.

2.3. Model establishment and analysis

According to relevant assumptions, this section carries out modeling and solving, and analyzes the influence of key parameters in the model on the optimal decision-making and profit of closed-loop supply chain members.

2.3.1. Manufacturer outsourcing mode (model O)

In model O, OEM first decides the quantity of new products $q_n$ and remanufacturing outsourcing costs $T_r$; TPR is paid according to OEM $T_r$ Then decide the amount of waste products recovered from consumers $q_r$. Both of them take profit maximization as their decision goal. The profit functions of OEM and TPR are:
\[ \Pi_M^* = (p_n - c_n)q_n + (p_r - T_r)q_r \] (1)

\[ \Pi_T^* = (T_r - c_r)q_r - \frac{1}{2} \eta q_r^2 \] (2)

According to the backward induction method, the optimal solution is obtained:

\[ q_n^* = \frac{(a-1)\alpha - \eta + (\alpha + \eta)c_n - ac_r}{2(a^2 - \alpha - \eta)} \]
\[ q_r^* = \frac{-ac_n + c_r}{2(a^2 - \alpha - \eta)} \]
\[ T_r^* = \frac{\alpha c_n + [2\alpha(1 - \alpha) + \eta]c_r}{2(a^2 - \alpha^2 + \eta)} \]

\[ p_n^* = \frac{1}{2} (1 + c_n) \]
\[ p_r^* = \frac{\alpha[a - \alpha^2 + \eta + \eta c_n - (a - 1) c_r]}{2(a^2 - \alpha^2 + \eta)} \]
\[ P_M^* = \frac{\alpha - \alpha^2 + \eta + (\alpha + \eta)c_n^2 + c_r^2 - 2c_n(a - \alpha^2 + \eta + ac_r)}{4(a^2 - \alpha^2 + \eta)} \]
\[ P_T^* = \frac{\eta(a c_n - c_r)^2}{8(a^2 - \alpha^2 + \eta)^2} \]

2.3.2. Unit authorized remanufacturing mode (model U)

In U model, OEM authorizes TPR patent license, and charges patent fee for every unit remanufactured product produced by TPR. TPR recycles waste products for remanufacturing and sells remanufactured products. First, OEM determines the number of new products \( q_n \) and patent license fee of remanufacturing unit \( f^U \), TPR paid according to OEM \( f^U \). Then decide the amount of waste products recovered from consumers \( q_r \). Both of them take profit maximization as their decision goal. The profit functions of OEM and TPR are:

\[ \Pi_M^U = (p_n - c_n)q_n + f^U q_r \] (3)
\[ \Pi_T^U = (p_r - c_r - f^U)q_r - \frac{1}{2} \eta q_r^2 \] (4)

According to the backward induction method, the optimal solution is obtained:

\[ q_n^U = \frac{\alpha(a - 2) - \eta + (2\alpha + \eta)c_n - ac_r}{2(a^2 - 2\alpha - \eta)} \]
\[ q_r^U = \frac{-ac_n + c_r}{2(a^2 - 2\alpha - \eta)} \]
\[ p_r^U = \frac{1}{2} (1 + c_n) \]
\[ f^U = \frac{1}{2} (\alpha - c_r) \]
\[ \Pi_M^U = \frac{\alpha(a - 2) + \eta + (2\alpha + \eta)c_n^2 + c_r^2 + 2c_n[a(a - 2) + \eta + ac_r]}{4(a^2 - 2\alpha - \eta)} \]
\[ \Pi_T^U = \frac{(2\alpha + \eta)(ac_n - c_r)^2}{8(a^2 - 2\alpha - \eta)^2} \]

2.3.3. Independent remanufacturing mode (model D)

In D model, OEM produces and sells new products, TPR recycles waste products and independently produces and sells remanufactured products. In which OEM decides the number of new products \( q_n \), TPR determines the quantity of waste products recovered from consumers \( q_r \). Both of them take profit maximization as their decision-making goal. The profit functions of OEM and TPR are:

\[ \Pi_M^D = (p_n - c_n)q_n \] (5)
\[ \Pi_T^D = (p_r - c_r)q_r - \frac{1}{2} \eta q_r^2 \] (6)

According to the backward induction method, the optimal solution is obtained:

\[ q_n^D = \frac{\alpha(a - 2) - \eta + (2\alpha + \eta)c_n - ac_r}{2(a^2 - 2\alpha - \eta)} \]
\[ q_r^D = \frac{\alpha[a(a - 2) - \eta - (2\alpha + \eta)c_n] + (4\alpha - \alpha^2 + 2\eta)c_r}{2(a^2 - 2\alpha - \eta)2\alpha + \eta)} \]
\[ p_n^D = \frac{\alpha(2\alpha + \eta + (2\alpha + \eta)c_n + ac_r)}{2(2\alpha + \eta)} \]
\[ p_r^D = \frac{\alpha[a + \eta][a(a - 2) - \eta - (2\alpha + \eta)c_n] + [(4\alpha - \alpha^2 + 2\eta)c_r]^2}{2(a^2 - 2\alpha - \eta)2\alpha + \eta)} \]
\[ \Pi_M^D = \frac{[\alpha(2\alpha + \eta) - \eta - (2\alpha + \eta)c_n + ac_r]^2}{4(a^2 - 2\alpha - \eta)2\alpha + \eta)} \]
\[ \Pi_T^D = \frac{\alpha[a(a - 2) - \eta - (2\alpha + \eta)c_n] + [(4\alpha - \alpha^2 + 2\eta)c_r]^2}{8(2\alpha + \eta)(2\alpha - \alpha^2 + \eta)^2} \]
Comparison of three remanufacturing modes

This section compares the equilibrium price, quantity and profit of closed-loop supply chain under three remanufacturing modes, and discusses the influence of OEM and TPR profit changes on OEM remanufacturing mode selection decision under different modes.

Inference 1 The relationship between the sales price and size of new products: \( p_{n*}^O = p_{n*}^U > p_{n*}^D \)

Inference 1 shows that the price of new products under independent remanufacturing mode is the lowest, while the prices under outsourcing remanufacturing model and unit authorized remanufacturing model are high and equal. The lowest price of new products in model D is mainly caused by market competition; In model O, OEM can control the quantity of remanufactured products through outsourcing costs; In model U, OEM charges TPR a unit license fee to adjust the quantity of remanufactured products, so OEM maintains an absolute dominant position in both models, thus keeping the retail price of new products unchanged.

Inference 2 The relationship between the sales price and size of remanufactured products: \( p_{r*}^U > p_{r*}^O > p_{r*}^D \)

Inference 2 shows that the price of remanufactured products is the lowest under the independent remanufacturing mode, which is caused by market competition; The price of remanufactured products under outsourcing remanufacturing mode is second, because OEM sells the remanufactured products of new products at the same time in model O, and remanufactured products take low price as competitive advantage compared with new products in market competition, so as to win more market share; While the price of remanufactured products is the highest in authorized mode, the main reason is that although OEM reduces the advantage of remanufacturing market through authorized remanufacturing, TPR is responsible for the sales of remanufactured products in this mode, and the impact on OEM market competition is greater than that of remanufactured products in outsourced remanufacturing mode. At this time, TPR can guarantee profits by increasing the price of remanufactured products.

Inference 3 The quantity of new products: \( q_{n*}^O < q_{n*}^U = q_{n*}^D \)

Inference 3 shows that the number of new products of OEM is the smallest in outsourcing remanufacturing mode and the larger in authorization mode, and the number of new products in unit authorization mode is equal to that in independent remanufacturing mode. In model O, OEM sells new products and remanufactured products at the same time, which is affected by brand effect. at this time, the sales of remanufactured products have the greatest impact on new product sales. In model U and model D, OEM and TPR compete with each other to sell new products and remanufactured products respectively. OEM can reduce the market of remanufactured products by increasing the number of new products, so the number of new products in model o is the lowest.

Inference 4 Quantity relationship of reproductions:

When \( 0 < \eta \leq \eta_1, q_{r*}^O > q_{r*}^D > q_{r*}^U \); when \( \eta > \eta_1, q_{r*}^D > q_{r*}^O > q_{r*}^U \), where \( \eta_1 = \frac{2a^3 + a^2(c_n - 2c_n - 3) + 2ac_n + \sqrt{a^3(1 + c_n^2) + 2a^2c_n(1 + 2a) - 4a^2c_n(1 + a)(1 + c_n) + 4a^2c_n^2(1 + \alpha)}}{2(a - c_n)} \)

Inference 4 shows that when \( 0 < \eta < \eta_1 \) time, the number of remanufactured products is the highest in the outsourcing remanufacturing mode and the lowest in the authorized remanufacturing mode. When \( \eta > \eta_1 \), the quantity of remanufactured products is the highest in independent remanufacturing mode and the lowest in unit authorized remanufacturing mode. In the market competition, OEM reduces the remanufacturing market by increasing the number of new products. Because the number of new products in model U is higher than that in model O, the increase of new products in model U poses a threat to the remanufacturing market. In addition, OEM can control the quantity of TPR remanufactured products through
unit licensing fees, so the quantity of remanufactured products in model U is always smaller than that in model O.

Inference 5 the size of OEM profits: \( \Pi^*_{M} > \Pi^*_{O} > \Pi^*_{U} \)

Inference 5 shows that OEM has the highest profit under outsourcing remanufacturing mode, followed by authorized remanufacturing mode and the lowest profit under independent remanufacturing mode. Firstly, it is proved that when remanufactured products do not enter the market, there are only new products produced by OEM in the market, and the new product income of OEM at this time is \( p_n - c_n = 1 - q_n - c_n \). OEM profit is \((1 - q_n - c_n)q_n\) and get the optimal solution \( q_n^* = \frac{1-c_n}{2}, \Pi_M^* = \frac{(1-c_n)^2}{4} \), compared to get \( \Pi_M^* - \Pi_M^* > 0 \). It shows that when OEM does not adopt outsourcing remanufacturing or unit authorized remanufacturing, the sales of remanufactured products in independent remanufacturing mode will directly damage OEM’s profits. In addition, because the price of remanufactured products is lower than that of new products, the number of new products will decrease, which will lead to a decrease in the profits of new products. However, under the authorized remanufacturing mode, although the appearance of remanufactured products will erode the market of new products, which will lead to the reduction of the price and quantity of new products, due to the protection of patent property rights, OEMs can charge TPR unit licensing fees through the unit licensing mode to increase their own profits. Under the outsourcing remanufacturing mode, OEM’s profits come from the sales of new products and remanufactured products. Although the quantity of new products will also be affected by the sales of remanufactured products, OEM can reduce and increase the marginal profits of remanufactured products by setting outsourcing prices, or directly use the increase in sales of remanufactured products to compensate for the decrease in sales of new products, so as to maximize their profits.

Inference 6 The relationship between TPR profits:

When \( c_r < \alpha \leq \alpha(\eta), \Pi^*_{O} > \Pi^*_{M} > \Pi^*_{U} \); when \( \alpha(\eta) < \alpha < 1, \Pi^*_{O} > \Pi^*_{M} > \Pi^*_{U} \), where \( \alpha(\eta) \) is the unique solution of \( 2\alpha(1-\alpha)^2 + \eta(1 - 2\alpha)=0, \alpha(\eta) \in (0.5,1) \).

Inference 6 shows that TPR has the highest profit in independent remanufacturing mode because TPR has full control of remanufacturing activities and obtains all the benefits of remanufacturing activities. When the consumer’s preference for reproductions is low, that is \( c_r < \alpha \leq \alpha(\eta) \). The threat of remanufactured products to new products is very low, and OEM will only charge TPR less licensing fees for remanufacturing, then \( \Pi^*_{O} > \Pi^*_{M} \); As consumers’ preference for reproductions increases, that is \( \alpha(\eta) < \alpha < 1 \). At that time, remanufactured products began to eat away at the sales of new products, and the profits of remanufactured products will be bigger and bigger. At this time, OEM will also participate in remanufacturing activities to gain the benefits of remanufactured products market. When TPR continues to pay unit licensing fees to OEMs and sell remanufactured products, OEMs tend to charge TPR high licensing fees.

2.5. OEM introduces retailer outsourcing remanufacturing mode

In reality, compared with OEM and TPR, retailers have more advantages in sales and recycling channels, that is, after manufacturers introduce retailers, they can reduce the difficulty of recycling TPR, that is, reduce the cost of recycling, so the cost of recycling efforts is \( C = \frac{1}{2}(\eta - \rho)q_r^2 \), where \( \eta > 0 \). As a scale parameter, it reflects the difficulty of recovery; \( 0 < \rho \leq \eta \), reflecting the degree of reduction of recycling difficulty.

In model S, OEM first decides the wholesale price of new products \( w_n \) and unit licensing fees outsourced by authorized retailers \( f^S \). Then the retailer decided to \( q_n \), the number of new products will be sold and the remanufacturing business will be outsourced by unit \( T_r \) to TPR, TPR finally determines the quantity of recycled waste products \( q_r \). All of them take their profit.
maximization as their decision-making goal, and the profit functions of OEM, retailer and TPR are as follows:

\[
\begin{align*}
\Pi_M^S &= (w_n - c_n)q_n + f^S q_r \\
\Pi_R^S &= (p_n - w_n)q_n + (p_r - T_r - f^S) q_r \\
\Pi_T^S &= (T_r - c_r)q_r - \frac{1}{2}(\eta - \rho)q_r^2
\end{align*}
\]

According to the backward induction method, the optimal solution is obtained:

\[
\begin{align*}
w_n^{S*} &= \frac{1}{2}(1 + c_n), f^{S*} = \frac{1}{2}(\alpha - c_r), T_r^{S*} = \frac{\alpha(\eta - \rho)c_n + [4\alpha(1 - \alpha) - 3(\eta - \rho)]c_r}{4(\alpha - \alpha^2 - \eta + \rho)}, \\
q_n^{S*} &= \frac{(\alpha - 1)\alpha - \eta + \rho + (\alpha + \eta - \rho)c_n - ac_r}{4(\alpha - \alpha^2 - \eta + \rho)}, q_r^{S*} = \frac{-\alpha c_n + c_r}{4(\alpha - \alpha^2 - \eta + \rho)}, \\
p_n^{S*} &= \frac{1}{4}(3 + c_n), p_r^{S*} = \frac{\alpha[3(\alpha^2 - \alpha - \rho\eta) - \eta c_n + (\alpha - 1)c_r]}{4(\alpha - \alpha^2 - \eta - \rho)}, \\
\Pi_M^{S*} &= \frac{\alpha - \alpha^2 + (\eta - \rho) + (\alpha + \eta - \rho)c_n^2 + c_r^2 - 2c_n(\alpha - \alpha^2 + \eta - \rho + \alpha c_r)}{8(\alpha - \alpha^2 + \eta - \rho)}, \\
\Pi_R^{S*} &= \frac{\alpha - \alpha^2 + (\eta - \rho) + (\alpha + \eta - \rho)c_n^2 + c_r^2 - 2c_n(\alpha - \alpha^2 + \eta - \rho + \alpha c_r)}{16(\alpha - \alpha^2 + \eta - \rho)}, \\
\Pi_T^{S*} &= \frac{(\eta - \rho)(ac_n - c_r)^2}{32(\alpha - \alpha^2 + \eta + \rho)^2}
\end{align*}
\]

Inference 7 comparison of OEM profit optimal models:

When \(0 < \rho < \rho_1, \Pi_M^{O*} > \Pi_M^{S*}\), when \(\rho_1 < \rho < \eta, \Pi_M^{S*} > \Pi_M^{O*}\), where

\[
\rho_1 = \frac{\alpha^4(1-2c_n)+\eta^2(1-2c_n)^2+ac_r^2+\alpha^2[(1-c_n)^2-c_r(2c_n+c_r)]}{\alpha-\alpha^2+\eta-2\alpha c_n(1-\alpha)-\eta c_n(2-c_n)+ac_n^2(1+\alpha)-4ac_n c_r+2c_r^2}
\]

Inference 7 shows that when \(0 < \rho < \rho_1, \eta - \rho\) large enough, the recycling of TPR in closed-loop supply chain is difficult enough, and the high recycling cost of TPR will affect its recycling enthusiasm. At this time, the introduction of retailers will damage the profits of OEMs, so manufacturers will choose the outsourcing remanufacturing mode of manufacturers; when \(\rho_1 < \rho \leq \eta, \eta - \rho\) small enough, the recycling cost saved by TPR can not only increase the profit of remanufacturing, but also increase the income of OEM. At this time, OEM introduces retailers and authorizes retailers to outsource, which can improve their own income \(\Pi_M^{S*} > \Pi_M^{O*}\).

Inference 8 TPR profit optimal model comparison; \(\Pi_T^{O*} > \Pi_T^{S*}\)

Inference 8 shows that after the manufacturer introduces retailers, TPR still acts as a foundry and can only obtain corresponding outsourcing income; In the independent remanufacturing mode, TPR has full control over the remanufacturing activities and gets all the profits, so the profit is the highest at this time.

### 3. Analysis

In order to verify the correctness of the model and inference and make further analysis, this paper uses Mathematica software to analyze the pricing decision under different remanufacturing modes and the relationship between members' profits in closed-loop supply chain. The values of parameters in the model are as follows: \(c_r = 0.2, c_n = 0.7, \alpha \in (0.5, 0.9), \eta \in (0.3,1)\), as shown in Fig. 2~5:

Fig.2 shows that the price of new products is the smallest in independent remanufacturing mode, and the price of new products is the largest and equal in manufacturer outsourcing remanufacturing and unit authorized remanufacturing mode, which verifies inference 1.Fig.3
shows that the price of remanufactured products is the lowest in independent remanufacturing mode, the next in outsourcing remanufacturing mode by manufacturers, and the highest in unit authorized remanufacturing mode, which verifies inference 2. Fig. 4 shows that the number of new products in independent remanufacturing mode is equal to that in unit authorized remanufacturing mode, and the number of new products in manufacturer outsourcing remanufacturing mode is the lowest, thus verifying inference 3. Fig. 5 shows that the number of remanufactured products is the lowest under the unit authorized remanufacturing mode, when $0 < \eta < \eta_1$ time, the number of remanufactured products is the highest under the outsourcing remanufacturing mode of manufacturers; when $\eta > \eta_1$, the number of remanufactured products is the highest in independent remanufacturing mode, thus verifying inference 4.

Analyze the difficulty of TPR recovery $\rho$. For the impact on OEM and TPR profits, the parameters in the model are selected: $c_r = 0.2$, $c_n = 0.7$, $\eta = 1$, $\alpha \in (0.5, 0.9)$, $\rho \in (0, 0.8)$, as shown in Fig. 6~7:

As can be seen from Fig. 6, in the manufacturer remanufacturing mode and the unit authorized remanufacturing mode, the profit of OEM follows the consumer’s preference for remanufacturing $\alpha$ increase with the increase of $\eta$. In the model D, the profit of OEM decreases with the increase of consumers' preference for remanufacturing $\alpha$ and OEM does not participate in remanufacturing activities, but TPR completely controls the remanufacturing activities, so when consumers' preference for remanufacturing increases, OEM's profits will be seriously hurt, so OEM's profits under this model are the lowest. In addition when $\rho$ less than a certain threshold, i.e. $0 < \rho < \rho_1$. The OEM will choose the manufacturer's outsourcing remanufacturing mode; when $\rho$ greater than a certain threshold, i.e. $\rho_1 < \rho \leq \eta$. OEM will choose retailer outsourcing remanufacturing mode to verify inference 7.

It can be seen from Fig. 7 that the profit of TPR follows $\alpha$. In the independent remanufacturing mode, TPR completely controls the remanufacturing activities, that is, the profit is the highest; When the consumer’s preference for remanufactured products is less than a certain threshold, that is, the consumer's acceptance of remanufactured products is low, and the threat of
remanufactured products to new products is very low. At this time, OEM will only charge TPR less licensing fees for remanufacturing, so TPR in unit remanufacturing mode will get higher benefits than TPR in manufacturer outsourcing mode. When the consumer's preference for remanufactured products is greater than a certain threshold, that is the consumer's acceptance of remanufactured products is higher, and the profits of remanufactured products will become larger and larger. At this time, OEM tends to authorize high licensing fees to TPR and participate in remanufacturing activities to gain the benefits of remanufactured products market.

4. Conclusions

In order to analyze the influence of manufacturing mode on market competition, this paper studies the remanufacturing mode of third-party remanufacturers on the basis of heterogeneous consumers, compares and analyzes the optimal pricing decisions of supply chain members under three remanufacturing models directly led by OEM, and obtains the optimal remanufacturing mode. Furthermore, the optimal remanufacturing model is compared with the model after introducing retailers, and the optimal remanufacturing mode adopted by OEMs is analyzed. The main conclusions are as follows:

1. In the closed-loop supply chain composed of OEM, TPR and consumers, the profit of OEM is the highest in the outsourcing remanufacturing mode of manufacturer, followed by the authorized remanufacturing mode of unit and the lowest in the independent remanufacturing mode of TPR.

2. After OEM introduces retailers, only when the recycling cost is low enough, that is, the difficulty of recycling is greatly reduced, can the introduction of retailers improve their own profits. Therefore, there is a threshold for the reduction of the difficulty of recycling. Only when this threshold is greater can manufacturers consider introducing retailers to increase their profits.

This paper discusses OEM's remanufacturing strategy based on certain assumptions, which has certain limitations, so the problems worthy of further study are as follows: (1) This paper only considers a single remanufacturing subject for recycling and remanufacturing, and can further study the optimal remanufacturing mode selection when manufacturers compete with TPR for recycling and remanufacturing; (2) Market demand is generally uncertain, so the pricing strategy of remanufacturing closed-loop supply chain considering uncertain demand is also the future research direction.

Acknowledgments

National Natural Science Foundation of China (71971134);
References


