

Dynamic Competition Analysis of Tourism City Advertising Investment based on Cournot Competition

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Abstract

The main aim of tourism city advertisement has changed from publicizing city to competing for market demand through characteristic attraction, and the competition among cities is a dynamic game process. Based on the classical Cournot competition model, a dynamic model of tourism city advertising investment competition is established by using difference equations. The equilibrium solution and dynamic process of the model are given, it is proved that the equilibrium solution exists and is unique, and the final adjusted investment level is the equilibrium quantity, no matter what the initial choice of the investment level of a tourism city is when it enters the advertising market, this is consistent with the classic Cournot competitive conclusion. Finally, the article proves that the change process of the amount of tourism city advertising investment is necessarily a monotonous process of change.

Keywords

Tourism City; Advertising Investment; Dynamic Competition; Equilibrium.

1. Introduction

The tourism city's publicity is increasingly strengthened, and the competition analysis of its advertising investment has been less. Such as Zhong bi-wei ZQ City for Tourism City brand communication status analysis and problem analysis, and targeted brand communication strategy optimization research [1]. Because the competitive process is a long-term dynamic game process, it needs to be discussed quantitatively in order to find its corresponding competitive mechanism and strategy, and the research in this area is even more insufficient. Chen Zhezhi introduced the supply chain management theory into the research on the development of low-carbon tourism, and used game analysis to study the competition and cooperation among the main bodies of the low-carbon tourism supply chain, to seek a scientific and reasonable low-carbon tourism supply chain optimization path [2], for our analysis provides a new perspective. Secondly, Wang Yu regards the local government and tourism service industry as a kind of tourism supply chain, from the perspective of supply chain coordination, this paper proposes an optimal mechanism design for the organization and management of the urban marketing and tourism service industries of tourist cities, so as to realize the overall profit maximization of urban marketing while local governments pursue the urban marketing, individual profit maximization in tourism services [3].

In the current situation of increasing competition, tourism city for the current competition between consumers formed by the intensification of inter-city tourism competition and "Traffic is King" in the emerging Internet red tourism city, such as the actual situation, the focus of city tourism publicity competition has changed from the "Single stage" of "City image publicity" to the "Improvement and innovation stage" of "City characteristics based on comprehensive

performance publicity”, to meet the needs of new markets. Therefore, the result of advertising investment is mainly to improve the quality of the final city, and then enhance the final tourism image in the market bargaining power, this kind of city local government and tourism service industry has become a kind of tourism supply chain, and the tourism competition among cities has evolved into the competition among the main bodies of the tourism supply chain. In the analysis, this kind of supply chain can be regarded as the independent behavior of the tourism city. In addition, the competition of city advertising investment is a dynamic game process, in which the equilibrium solution of the dynamic model and the dynamic change of R&D competition sequence are analyzed. Clearly, the current literature does not address these issues. Based on the classic Cournot model of competition, this paper aims to improve the quality of cities and increase the market demand for publicity input, this paper analyzes the dynamic process of advertising investment competition in two competitive cities and the problems of its equilibrium solution by means of recurrence relation method, lay the foundation for further analysis and discussion.

2. The Introduction of Competition Model of Advertising Investment in Two Cities

In the case of a regional tourism market in which there are two tourism cities of comparable strength 1 and 2 competing for tourism promotion, each city acting on the actions of its opponent and assuming that the opponent continues to act in this manner, to make their own advertising investment decision. Each city according to each other’s input strategy, constantly adjust their choice strategy. The two tourism cities provide the same publicity model, showing constant economies of scale, the same unit cost c_0 and for investment. The first advertisement investment selected by the two manufacturers is respectively sum x_1 and x_2 and the average market demand generated by the advertisement investment of the tourism market unit in the region is β , which reflects the market benefit of the advertisement investment, generally from the advertising input in the market for the incremental consumption to obtain. Assuming that the tourism market is a rigid demand and the potential total publicity-driven demand is relatively constant, the tourism consumption demand generated by the input of $x(x = x_1 + x_2)$ advertising quantity is $q = q_1 + q_2 = \beta x = \beta(x_1 + x_2)$. In addition, suppose two tourist cities face the same linear demand function as follows:

$$P = a - b(q_1 + q_2) = a - b\beta(x_1 + x_2) \quad (1)$$

This hypothesis is consistent with the idea of literature [4]. For the tourism city that propagandizes itself vigorously, its variable cost is not only the relevant tourism investment, but also the advertisement investment expense. For their own aspects of tourism investment will vary from city to city, this paper mainly studies the latter situation.

The two cities compete through advertising to maximise profits:

$$\max[a - b\beta(x_1 + x_2) - c_0]\beta x_i - x_i \quad (i = 1, 2) \quad (2)$$

Make the profit is zero, available $x_1 + x_2 = \frac{(a - c_0)\beta - 1}{b\beta^2}$, remember $d = \frac{(a - c_0)\beta - 1}{b\beta^2}$, d is on behalf of this kind of tourism market advertising investment capacity.

When there is only one tourism city in the tourism market, the profit maximization investment in advertising is $x = \frac{(a-c_0)\beta-1}{2b\beta^2} = \frac{d}{2}$, that is, the optimal advertising investment is half of the market capacity. When two cities compete for advertisement, the advertising investment of City 1 is x_1 , then the optimal advertising competition investment of City 2 is $x_2 = \frac{(a-c_0)\beta-1}{2b\beta^2} - \frac{x_1}{2} = \frac{d-x_1}{2}$, which is half of the remaining market capacity.

If the advertising investment of city 1 decreases Δx_1 , that is, the advertising investment of City 1 becomes $x'_1 = x_1 - \Delta x_1$, then the optimal advertising investment of City 2 should satisfy:

$$x'_2 = \frac{(a-c_0)\beta-1}{2b\beta^2} - \frac{1}{2}(x_1 - \Delta x_1) = \frac{d-x_1}{2} + \frac{1}{2}\Delta x_1 = x_2 + \frac{1}{2}\Delta x_1 \quad (3)$$

Suppose that tourism city 1 increases its advertising investment by Δx_1 , similarly it can be proved that there must be $x'_2 = x_2 - \frac{1}{2}\Delta x_1$.

It can be obtained that when city 1 increases (or decreases) Δx_1 , city 2 decreases (or increases) $\Delta x_1/2$, and when City 2 increases (or decreases) Δx_2 , city 1 will spend less (or more) $\Delta x_2/2$ on advertising.

3. The Dynamic Process of the Advertising Investment Competition Model in Two Tourism Cities

Assuming that the first tourism city (set as City 1) enters the tourism promotion market for the first time, the amount of advertising investment chosen according to the profit maximization principle is as follows:

$$x_{1(1)} = \frac{(a-c_0)\beta-1}{2b\beta^2} = \frac{d}{2} \quad (4)$$

The number of City 2 inputs into the tourism promotion market will be adjusted from zero to:

$$x_{2(1)} = \frac{(a-c_0)\beta-1}{2b\beta^2} - \frac{1}{2} \cdot \frac{(a-c_0)\beta-1}{2b\beta^2} = \frac{d}{4} \quad (5)$$

The increase of investment is $\frac{d}{4}$, then, the investment of City 1 will decrease $\frac{1}{2} \cdot \frac{d}{4}$, that is, the optimal investment of City 1 will be adjusted to $x_{1(2)} = \frac{d}{2} - \frac{1}{2} \cdot \frac{d}{4}$. City 2 will increase its advertising by $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{d}{4} = \left(\frac{1}{4}\right)^2 d$, or $x_{2(2)} = \frac{d}{4} + \left(\frac{1}{4}\right)^2 d$. Similar analysis, two tourism cities next adjustment of advertising investment were:

$$x_{1(3)} = \frac{d}{2} - \frac{1}{2} \cdot \frac{1}{4}d - \frac{1}{2} \left(\frac{1}{4}\right)^2 d, \quad x_{2(3)} = \frac{d}{4} + \left(\frac{1}{4}\right)^2 d + \left(\frac{1}{4}\right)^3 d \tag{6}$$

After n times of adjustment, the tourism advertising investment of Cities 1 and 2 will be adjusted as follows:

$$\begin{aligned} x_{1(n)} &= \frac{d}{2} - \frac{1}{2} \cdot \frac{1}{4}d - \frac{1}{2} \left(\frac{1}{4}\right)^2 d - \dots - \frac{1}{2} \left(\frac{1}{4}\right)^{n-1} d \\ x_{2(n)} &= \frac{d}{4} + \left(\frac{1}{4}\right)^2 d + \left(\frac{1}{4}\right)^3 d + \dots + \left(\frac{1}{4}\right)^n d \end{aligned} \tag{7}$$

After numerous adjustments, tourism cities 1 and 2 will be adjusted to the balanced number of advertising investment x_1 and x_2 , they are:

$$\begin{aligned} x_1^* &= \lim_{n \rightarrow \infty} x_{1(n)} = \frac{d}{2} - \frac{d}{2} \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n = \frac{d}{2} - \frac{d}{2} \cdot \frac{1}{3} = \frac{d}{3} \\ x_2^* &= \lim_{n \rightarrow \infty} x_{2(n)} = \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n d = \frac{d}{3} \end{aligned} \tag{8}$$

(8) formula shows that when the first city enters the tourism advertising market for the first time and chooses the advertising investment under the condition of maximizing profit, the equilibrium quantity of the tourism advertising investment of the two cities is equal, which is $\frac{1}{3}$ of the market capacity.

However, the reality is that the first tourism city to enter the tourism market for the first time the amount of investment is arbitrary choice, different choice equilibrium solution may be different. The following analysis of the first entry into the market under the conditions of arbitrary initial selection of the equilibrium solution and dynamic change process.

Assuming that city 1 enters the tourism publicity market for the first time the amount of advertising investment is $x_{1(1)}$, then the Second City 2 enters the tourism publicity market for the first time the amount of advertising investment is $x_{2(1)} = \frac{d}{2} - \frac{1}{2}x_{1(1)}$, and then, the amount of tourism advertisement in City 1 will be adjusted to $x_{1(2)} = \frac{d}{2} - \frac{1}{2}x_{2(1)}$, and the investment of tourism advertisement in City 2 will be adjusted to $x_{2(2)} = \frac{d}{2} - \frac{1}{2}x_{1(2)}$. After m adjustments, the investment of tourism advertisement in City 1 and City 2 will be:

$$x_{1(m)} = \frac{d}{2} - \frac{1}{2}x_{2(m-1)}, \quad x_{2(m)} = \frac{d}{2} - \frac{1}{2}x_{1(m)} \tag{9}$$

The following analysis is done in vector form with the help of difference equations. (9) can be written as:

$$\begin{bmatrix} x_{1(m)} \\ x_{2(m)} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ -\frac{1}{2} & 0 \end{bmatrix} \begin{bmatrix} x_{1(m)} \\ x_{2(m)} \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1(m-1)} \\ x_{2(m-1)} \end{bmatrix} + \begin{bmatrix} \frac{d}{2} \\ \frac{d}{2} \end{bmatrix} \tag{10}$$

From (10):

$$\begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} x_{1(m)} \\ x_{2(m)} \end{bmatrix} = \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1(m-1)} \\ x_{2(m-1)} \end{bmatrix} + \begin{bmatrix} \frac{d}{2} \\ \frac{d}{2} \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} x_{1(m)} \\ x_{2(m)} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}^{-1} \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_{1(m-1)} \\ x_{2(m-1)} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & 1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{d}{2} \\ \frac{d}{2} \end{bmatrix} \tag{12}$$

$$= \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & \frac{1}{4} \end{bmatrix} \begin{bmatrix} x_{1(m-1)} \\ x_{2(m-1)} \end{bmatrix} + \begin{bmatrix} \frac{d}{2} \\ \frac{d}{4} \end{bmatrix}$$

If $x_{(m)} = \begin{bmatrix} x_{1(m)} \\ x_{2(m)} \end{bmatrix}$, $D = \begin{bmatrix} 0 & -\frac{1}{2} \\ 0 & \frac{1}{4} \end{bmatrix}$, $H = \begin{bmatrix} \frac{d}{2} \\ \frac{d}{4} \end{bmatrix}$, then (12) is:

$$x_{(m)} = Dx_{(m-1)} + H$$

That is:

$$\begin{aligned} x_{(m)} &= D^{m-1}x_{(1)} + D^{m-2}E + D^{m-3}E + \Lambda + D^2E + DE + E \\ &= D^{m-1}x_{(1)} + \sum_{i=0}^{m-2} D^i H \end{aligned} \tag{13}$$

Easy to prove $D^i = \begin{bmatrix} 0 & -\frac{1}{2} \left(\frac{1}{4}\right)^{i-1} \\ 0 & \left(\frac{1}{4}\right)^i \end{bmatrix}$, thus having:

$$D^{m-1} = \begin{bmatrix} 0 & -\frac{1}{2}\left(\frac{1}{4}\right)^{m-2} \\ 0 & \left(\frac{1}{4}\right)^{m-1} \end{bmatrix} \tag{14}$$

$$\begin{aligned} \sum_{i=0}^{m-2} D^i &= I + \sum_{i=1}^{m-2} D^i = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \sum_{i=1}^{m-2} \begin{bmatrix} 0 & -\frac{1}{2}\left(\frac{1}{4}\right)^{i-1} \\ 0 & \left(\frac{1}{4}\right)^i \end{bmatrix} \\ &= \begin{bmatrix} 1 & -\frac{2}{3}\left[1 - \left(\frac{1}{4}\right)^{m-2}\right] \\ 0 & 1 + \frac{1}{3}\left[1 - \left(\frac{1}{4}\right)^{m-2}\right] \end{bmatrix} \end{aligned} \tag{15}$$

$$\sum_{i=0}^{m-2} D^i H = \begin{bmatrix} 1 & -\frac{2}{3}\left[1 - \left(\frac{1}{4}\right)^{m-2}\right] \\ 0 & 1 + \frac{1}{3}\left[1 - \left(\frac{1}{4}\right)^{m-2}\right] \end{bmatrix} \begin{bmatrix} \frac{d}{2} \\ \frac{d}{4} \end{bmatrix} = \begin{bmatrix} \frac{d}{3} + \frac{1}{6}\left(\frac{1}{4}\right)^{m-2} d \\ \frac{d}{3} - \frac{1}{12}\left(\frac{1}{4}\right)^{m-2} d \end{bmatrix} \tag{16}$$

Thus, (14) can be transformed into:

$$\begin{aligned} x_{(m)} &= D^{m-1} x_{(1)} + \sum_{i=0}^{m-2} D^i H \\ &= \begin{bmatrix} 0 & -\frac{1}{2}\left(\frac{1}{4}\right)^{m-2} \\ 0 & \left(\frac{1}{4}\right)^{m-1} \end{bmatrix} \begin{bmatrix} x_{1(1)} \\ x_{2(1)} \end{bmatrix} + \begin{bmatrix} \frac{d}{3} + \frac{1}{6}\left(\frac{1}{4}\right)^{m-2} d \\ \frac{d}{3} - \frac{1}{12}\left(\frac{1}{4}\right)^{m-2} d \end{bmatrix} \\ &= \begin{bmatrix} -\frac{1}{2}\left(\frac{1}{4}\right)^{m-2} x_{2(1)} + \frac{d}{3} + \frac{1}{6}\left(\frac{1}{4}\right)^{m-2} d \\ \left(\frac{1}{4}\right)^{m-1} x_{2(1)} + \frac{d}{3} - \frac{1}{12}\left(\frac{1}{4}\right)^{m-2} d \end{bmatrix} \end{aligned} \tag{17}$$

By substituting $x_{2(1)} = \frac{d}{2} - \frac{1}{2}x_{1(1)}$ into (17), we can get the dynamic adjustment model of the competition of tourism advertising in two cities as follows:

$$\begin{aligned}
x_{1(m)} &= -\frac{1}{2}\left(\frac{1}{4}\right)^{m-2}\left(\frac{d}{2}-\frac{1}{2}x_{1(1)}\right)+\frac{d}{3}+\frac{1}{6}\left(\frac{1}{4}\right)^{m-2}d \\
&= \left(\frac{1}{4}\right)^{m-1}\left(x_{1(1)}-\frac{d}{3}\right)+\frac{d}{3} \\
x_{2(m)} &= \left(\frac{1}{4}\right)^{m-1}\left(\frac{d}{2}-\frac{1}{2}x_{1(1)}\right)+\frac{d}{3}-\frac{1}{12}\left(\frac{1}{4}\right)^{m-2}d \\
&= \frac{1}{2}\left(\frac{1}{4}\right)^{m-1}\left(\frac{d}{3}-x_{1(1)}\right)+\frac{d}{3}
\end{aligned} \tag{18}$$

Let the number of times of advertising game between the two cities be infinite, that is, $m \rightarrow \infty$, if the limit of $x_{1(m)}$ and $x_{2(m)}$ determined by the above two formulas exists, then the equilibrium solution exists, and the limit value is the equilibrium price.

$$\begin{aligned}
\lim_{m \rightarrow \infty} x_{1(m)} &= \lim_{m \rightarrow \infty} \left\{ \left(\frac{1}{4}\right)^{m-1} \left(x_{1(1)} - \frac{d}{3}\right) + \frac{d}{3} \right\} = \left(x_{1(1)} - \frac{d}{3}\right) \lim_{m \rightarrow \infty} \left(\frac{1}{4}\right)^{m-1} + \frac{d}{3} = \frac{d}{3} \\
\lim_{m \rightarrow \infty} x_{2(m)} &= \lim_{m \rightarrow \infty} \left\{ \frac{1}{2} \left(\frac{1}{4}\right)^{m-1} \left(\frac{d}{3} - x_{1(1)}\right) + \frac{d}{3} \right\} \\
&= \frac{1}{2} \left(\frac{d}{3} - x_{1(1)}\right) \lim_{m \rightarrow \infty} \left(\frac{1}{4}\right)^{m-1} + \frac{d}{3} = \frac{d}{3}
\end{aligned} \tag{19}$$

This shows that the equilibrium solution of the price model exists and is unique regardless of the first city's investment when it first enters the tourism advertising market, and the final adjusted advertising investment is the equilibrium quantity, both are $\frac{1}{3}$ of market capacity, which is consistent with the classic Cournot competition conclusion.

4. The Monotony of the Tourism Advertisement Investment Sequence in the Two Cities

The following continues to analyze the two cities tourism advertising competition dynamic change process, in order to find out the law of change. From (18):

$$x_{1(m+1)} = \left(\frac{1}{4}\right)^m \left(x_{1(1)} - \frac{d}{3}\right) + \frac{d}{3}, \quad x_{2(m+1)} = \frac{1}{2} \left(\frac{1}{4}\right)^m \left(\frac{d}{3} - x_{1(1)}\right) + \frac{d}{3} \tag{20}$$

Therefore, there are:

$$x_{1(m+1)} - x_{1(m)} = \left[\left(\frac{1}{4}\right)^m - \left(\frac{1}{4}\right)^{m-1} \right] \left(x_{1(1)} - \frac{d}{3}\right) = 3 \left(\frac{1}{4}\right)^m \left(\frac{d}{3} - x_{1(1)}\right)$$

$$x_{2(m+1)} - x_{2(m)} = \frac{1}{2} \left[\left(\frac{1}{4} \right)^m - \left(\frac{1}{4} \right)^{m-1} \right] \left(\frac{d}{3} - x_{1(1)} \right) = \frac{3}{2} \left(\frac{1}{4} \right)^m \left(x_{1(1)} - \frac{d}{3} \right) \quad (21)$$

Obviously, the value of the initial choice $x_{1(1)}$ of the first city has an effect on the rule of the change of the investment of the two cities, when the value is less than the equilibrium quantity, that is, $x_{1(1)} < \frac{d}{3}$, obtained from (21):

$$x_{1(m+1)} - x_{1(m)} > 0, x_{2(m+1)} - x_{2(m)} < 0 \quad (22)$$

Therefore, when the investment level of advertisement is less than the equilibrium quantity when the first city enters the tourism publicity market, the investment level of the first city will increase monotonously, the second city's advertising investment level will be strictly monotonous decline.

Similarly, when the initial advertising investment of the first city is greater than the equilibrium amount, that is, $x_{1(1)} > \frac{d}{3}$, we get:

$$x_{1(m+1)} - x_{1(m)} < 0, x_{2(m+1)} - x_{2(m)} > 0 \quad (23)$$

That is, when the advertising investment level of the first city is larger than the equilibrium quantity, the advertising investment level of the first city will decrease monotonously, the second city's advertising investment level will be strictly monotonous.

When the initial advertising investment of the first city is equal to the equilibrium amount, or $x_{1(1)} = \frac{d}{3}$, it obtained:

$$x_{1(m+1)} = x_{1(m)} = \frac{d}{3}, x_{2(m+1)} = x_{2(m)} = \frac{d}{3} \quad (24)$$

This shows that when the advertising investment of the first city entering the tourism publicity market is an equilibrium quantity, the advertising investment level of the two cities remains unchanged and remains at the equilibrium quantity level, in other words, there is no dynamic change process in the investment of tourism advertising in the two cities, and it reaches the equilibrium state at the beginning.

5. Conclusion

This paper gives a dynamic change process model of advertising investment in two tourism cities under the given demand function, based on the analysis of the equilibrium solution and the dynamic process of price series of the competition model of advertising investment in two tourism cities under the same unit cost, the following conclusions can be drawn:

(1) Under the condition of two competitive cities, no matter what the initial choice of the first city is, the equilibrium solution of the competition model exists and is unique.

(2) when the initial investment level of the first city is $\frac{d}{3}$, the competition between the two cities will reach equilibrium at the beginning. When the initial advertising level deviates from the equilibrium quantity, the equilibrium process of advertising competition between the two cities is an infinite process, and two city's advertisement investment adjustment reverse change. The model is based on the assumption of Cournot competition. In fact, the advertising cost and the average market demand generated by the unit advertising input are not the same in both cities, a regional tourism promotion market is not limited to two cities. In addition, we did not further analyze and discuss the β value of this average requirement. Therefore, for the different cost conditions and multi-city tourism advertising competition in the dynamic process of change, etc.

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