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Forecasting Value-at-Risk of European Union Allowance Futures with Time-varying Risk Aversion

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Abstract

According to the GARCH model and mixed frequency data sampling (MIDAS) structure, we propose the GJR-GARCH-MIDAS-RA model to forecast the value-at-risk (VaR) of EUA futures under the condition of time-varying risk aversion (RA). The model that we proposed incorporates RA into the EUA futures volatility research framework using the MIDAS structure and fully accounts for the leverage effect of EUA futures. The empirical analysis based on monthly time-varying risk aversion (RA) index data and daily EUA futures closing price data shows that RA has a significant negative impact on EUA futures volatility and that the GJR-GARCH-MIDAS-RA model performs better in forecasting the VaR of EUA futures than many other competing models.

Keywords

EUA Futures; Time-varying Risk Aversion; Volatility Timing; GJR-GARCH-MIDAS Model; Leverage Effect.

1. Introduction

With the iterative improvement of basic policy systems such as monitoring, reporting and verification under the EU ETS framework, the coverage of carbon trading has expanded, but the total amount of carbon allowance has been continuously reduced. The trading conditions of the EU carbon market have become more complicated. As the main component of carbon derivatives for investors to invest profitably and diversify investment risks, European Union Allowance (EUA) futures has experienced dramatic volatility during the 2008-2009 subprime mortgage crisis, the 2011-2012 European debt crisis and 2020 COVID-19 outbreak, which increased the possibility of systemic risk in the EUA futures market (Lutz et al., 2013). In addition, when the outbreak of the Russo-Ukrainian war on February 24, 2022, the EUA futures price has fallen by 38.78% in less than half a month, and then risen by 31.25% in 3 days, which fluctuated sharply. Therefore, it is of great practical significance to effectively measure the risk of EUA futures price fluctuations to help individuals and institutional investors effectively avoid investment risks and provide more effective risk management tools for emission control enterprises.

This paper innovates and contributes in filling some existing gaps in the literature in the following directions. First, we investigate the relationship between time-varying risk aversion (RA) and forecasting VaR for EUA futures. Second, based on the GARCH model and mixed frequency data sampling (MIDAS) structure, we propose the GJR-GARCH-MIDAS-RA model to forecast the value-at-risk (VaR) of EUA futures under the condition of RA. Third, the model that we proposed incorporates RA into the EUA futures volatility research framework using the MIDAS structure and fully accounts for the leverage effect of EUA futures. The empirical analysis based on monthly time-varying RA index data and daily EUA futures closing price data shows that RA has a significant negative impact on EUA futures volatility and that the GJR-GARCH-MIDAS-RA model performs better in forecasting the VaR of EUA futures than many other competing models (GARCH, GAGRCH-MIDAS, GARCH-MIDAS-RA, GJR-GARCH-MIDAS).

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The research that we make offers an effective method for modeling and forecasting the volatility of EUA futures, and supplies theoretical support for investors and emission control enterprises to effectively avoid the investment risk of EUA futures. In addition, the GJR-GARCH-MIDAS-RA model depicts the VaR of EUA futures well, which brings a useful reference for optimizing the market risk management mechanism, and has certain theoretical and practical significances.

The remainder of the paper is organized as follows: In Section 2, we present the related studies. Section 3 describes the materials and methods. Section 4 provides the empirical results, and Section 5 concludes.

2. Related Studies

The risk of investing in EUA futures can be quantified using Value at Risk (VaR). For a given time horizon t and confidence level p, the VaR is the loss in market value over the time horizon t that is exceeded with probability $1-p$ (Duffie and Pan, 1997). The essence of VaR is the quantile of loss distribution, and VaR has become a standardized risk measurement tool for financial institutions due to its simple structure, convenient computation and succinct generalization. Feng et al. (2012) utilize the extreme value theory and GARCH model to forecast the VaR of EUA futures, they demonstrate that the VaR based on the GARCH model and extreme value theory can effectively forecast the VaR of EUA futures. Zhu et al. (2019) apply different timescales to measure the risk of European carbon market, they illustrate that multiscale VaR model can effectively reduce the influence of extreme events compared with the traditional VaR framework of single timescale. Although the methods used in the above literature can forecast the VaR of carbon market, they fail to consider the impact of exogenous variables on carbon price fluctuations and carbon market risks caused by the price fluctuations. Chevallier (2011), Koch et al. (2014), Zhu et al. (2015) and Huang et al. (2022) demonstrate that carbon price fluctuations are closely related to exogenous variables. Jiao et al. (2018) apply an economic state-dependent approach that incorporates information about exogenous variables to evaluate VaR of carbon market, and illustrate that the model incorporating exogenous variables could improve the forecasting accuracy of VaR for carbon market.

In recent years, many scholars have taken time-varying Risk Aversion (RA) index as an exogenous variable into account to study the financial market volatility. The construction of RA index is based on a dynamic no-arbitrage asset pricing model consisting of six observable financial variables including the term spread, credit spread, detrended dividend yield, realized and risk-neutral equity return variances, and realized corporate bond return variance, which are extracted from direct price information on corporate bonds and equity markets in the US financial markets by Bekaert et al. (2022). But what needs to be pointed out here is that the secure use of the no-arbitrage asset pricing model assumes that the so-called epistemological puzzle of the continuity assumption (Walter, 2021) is solved in the case of EU allowances. The RA index, constructed by Bekaert et al. (2022), is utility-based, reflecting the time-varying relative risk aversion coefficient of the representative agent in a generalized habit-like model with preference shocks, and has been extensively studied for financial market volatility. Thus, the RA index distinguishes time-varying economic uncertainty (the amount of risk) from time-varying risk aversion (the price of risk), which considers the impact of different economic operating environments on investor sentiment, and become an unbiased measure of RA. There are lots of literature show that RA would affect financial market volatility. Some studies have shown that RA contains useful information for predicting fluctuations in EUA futures. Demirer et al. (2018) found that global RA is an important determinant of international equity market correlation. Xu (2019) shows that time-varying global RA can explain the dynamics of global equity returns. Demirer et al. (2022) demonstrate that RA could forecast the crude oil market

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volatility. Wu et al. (2022a) find that RA has a significant negative impact on the long-term volatility of EUA futures, and incorporating RA into the model can improve the model's prediction ability on the price volatility of EUA futures.

At the same time, some literature suggest that RA affects the volatility of traditional financial markets. Demirer et al. (2018; 2019; 2022) show that RA has a significant impact on returns and volatility of emerging equity markets, gold markets and crude oil markets, and RA contains important predictive information. Lee and Yoon (2020) investigate the dynamic spillover effect between the EUA futures market and the Brent crude oil market, they find that there is a strong volatility spillover effect between the two markets. The research of Yuan and Yang (2020) shows that financial market uncertainty has a large asymmetric risk spillover effect on the carbon market, and when systematic events occur, the uncertainty in the equity market has a larger risk spillover effect on the carbon market than the crude oil market. In addition, Nie et al. (2022), in their study of the spillover effects of price volatility between renewable energy stocks, technology stocks, oil futures and carbon allowances, find that renewable energy stocks have a significant spillover effect on carbon prices in the short term, and other factors such as energy prices, climate and policies may have a greater impact on the price of carbon allowances in the longer time dimension. It is not difficult to see that RA has large impact on the volatility of financial markets, while the volatility of financial markets and carbon market fluctuations are closely related. Therefore, to some extent, RA may also have an impact on carbon market volatility and its VaR level.

Motivated by above insights, few studies considering RA as an exogenous variable in their models to study its impact on the prediction of EUA futures, so there are still gaps in research on VaR of the EUA futures considering the exogenous variable (RA). Therefore, this paper aims to investigate the impact of RA on EUA futures price volatility and forecast the VaR of EUA futures. As the RA index constructed by monthly financial variables includes not only basic information of daily RA, but also the fundamental information of monthly macroeconomics, which can better reflect the changes of investors' risk aversion, so we choose the monthly RA index as the proxy variable of investors' RA. However, RA is a low-frequency (monthly) variable, which is inconsistent with the frequency of daily EUA futures returns, which poses a challenge to incorporate RA into the modeling of EUA futures volatility. The traditional approach is to convert high-frequency financial data into low frequency. Obviously, this method ignores the important structural features of the sample data, which will lead to the loss of high-frequency information, resulting in poor forecasting performance.

Inspired by the mixed data sampling (MIDAS) method of Ghysels et al. (2007), Engle et al. (2013) extend the traditional GARCH model into the GARCH-MIDAS model by decomposing conditional volatility into two components: a (high frequency) short-term component and a (low frequency) long-term component, where the long-term component is modeled by the MIDAS method, which allows the direct introduction of low-frequency explanatory variables (realized volatility and macroeconomic variables) to model and forecast volatility. GARCH-MIDAS model makes full use of high (low) frequency information, avoids the loss of information and improves the accuracy of volatility forecasts. Therefore, GARCH-MIDAS model has received a lot of attention in the literature. Liu et al. (2021) apply the GARCH-MIDAS model with economic policy uncertainty to model and forecast the volatility of EUA futures, the empirical results show that the GARCH-MIDAS model performs a better out-of-sample predictive ability compared with traditional GARCH-type models. Dai et al. (2022) construct GARCH-MIDAS-EUEPU and GARCH-MIDAS-GEPU models to investigate the impact of EU and global economic policy uncertainty on the volatility of EU carbon market. The results show that both European and global economic policy uncertainty will aggravate the long-term volatility of European carbon spot returns. Wu et al. (2022b) apply EGARCH-MIDAS model incorporating climate policy uncertainty to model and forecast the EUA futures, and they demonstrate that the EGARCH-MIDAS model with

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exogenous variable could improve the forecasting ability for EUA futures volatility. Therefore, given the low frequency (monthly) characteristics of the exogenous variable RA index, the MIDAS structure is required to include it in the study of its impact on high frequency time series. Despite its empirical successes, the GARCH-MIDAS model has certain shortcomings. For example, it cannot capture the asymmetry (leverage effect) of financial asset returns. We therefore extend the GARCH-MIDAS model to a GJR-GARCH-MIDAS model to capture the leverage effect of EUA futures returns (Conrad et al., 2012). Meanwhile, Rannou and Barneto (2016) showed that there is a leverage effect in the process of EUA futures price volatility and used a GJR-GARCH model to portray the asymmetric phenomenon of EUA futures price volatility, so this paper constructs a GJR-GARCH-MIDAS model and incorporates the RA index into the framework of this model to model and forecast the VaR of EUA futures.

3. Methodology

3.1. GJR-GARCH-MIDAS Model

GARCH model is a single component model, which can only model and forecast the volatility based on historical returns information. Studies have shown that long-term volatility in financial markets is not only affected by historical volatility, but also by macroeconomic information (Engle et al., 2013; engle and Rangel,2008), but the macroeconomic variables are usually sampled at a lower frequency (e.g. monthly) than the daily returns, so it is not feasible to introduce it into standard GARCH models that based on co-frequency data. In order to overcome this problem, Engle et al. (2013) propose the GARCH-MIDAS model, which can easily introduce macroeconomic variables (data that are sampled at a different frequency from the daily returns). Meanwhile, it has been well documented in the literature that financial assets returns usually show the characteristics of volatility asymmetry (leverage effect). But the GARCH-MIDAS model could not capture the leverage effect of returns on financial assets. Therefore, we apply the GJR-GARCH-MIDAS to capture the leverage effect of EUA futures returns and to forecast the VaR of EUA futures. The specification of GJR-GARCH-MIDAS model is given by:

$$r_{i,t} = \mu + \sigma_{i,t}\epsilon_{i,t} \tag{1}$$

$$\sigma_{i,t}^2 = \tau_t \times g_{i,t} \tag{2}$$

$$\varepsilon_{i,t} \mid I_{i-1,t} \sim i.i.d.\ N(0,1) \tag{3}$$

where $r_{i,t}$ is the logarithmic of the return on day i of the month t. It can be seen from Eq.(2) that the conditional variance $\sigma_{i,t}^2$, is multiplicatively decomposed into a short-term (high-frequency, daily) volatility component $g_{i,t}$, and a long-term (low-frequency, monthly) volatility component τ_t. The short-term volatility component $g_{i,t}$, follows a GJR-GARCH (1,1) process:

$$g_{i,t} = \left(1 - \alpha - \beta - \frac{\gamma}{2}\right) + \left(\alpha + \gamma N_{i-1,t}\right)\frac{\left(r_{i-1,t} - \mu\right)^2}{\tau_t} + \beta g_{i-1,t} \tag{4}$$

ISSN: 2692-7608 DOI: 10.6981/FEM.202311_4(11).0040

where the parameter γ, is the coefficient of leverage effect, and $\gamma > 0$ shows that there is a leverage effect of EUA futures returns. In order to ensure non-negativity and stationarity of the short-term component $g_{i,t}$, we impose some assumptions that $\alpha > 0$, $\beta > 0$, and $\alpha + \beta + \frac{\gamma}{2} < 1$. $N_{i-1,t}$ is the indicative variable of $\sigma^2_{i-1,t}\epsilon_{i-1,t}$.

$$N_{i-1,t} = \begin{cases} 0, & \sigma^2_{i-1,t}\epsilon_{i-1,t} < 0 \\ 1, & \sigma^2_{i-1,t}\epsilon_{i-1,t} > 0 \end{cases} \tag{5}$$

The long-term volatility component, τ_t, is specified by smoothing realized volatility (RV) in the spirit of MIDAS regression:

$$log(\tau_t) = m + \theta_1 \sum_{k=1}^{K} \varphi_k(\omega_1) log(RV_{t-k}) + \theta_2 \sum_{k=1}^{K} \varphi_k(\omega_2) log(RA_{t-k}) \tag{6}$$

where K is the number of periods over which we smooth the volatility, $\varphi_k(\cdot)$ is a non-negative weighting function and RV_t is the monthly RV, which is defined as:

$$RV = \sum_{i=1}^{N_t} r^2_{i,t} \tag{7}$$

where N_t is the number of trading days in month t. Following the practice of Engle et al. (2013), Asgharian et al. (2016) and Li et al. (2020), we choose one-parameter Beta polynomial as the weighting function $\varphi_k(\cdot)$, which is given by:

$$\varphi_k(\omega) = \frac{(1 - k/K)^{\omega-1}}{\sum_{j=1}^{K}(1 - j/K)^{\omega-1}} \tag{8}$$

where K is the number of MIDAS lags, and $\sum_{k=1}^{K} \varphi_k(\omega) = 1$.

3.2. Benchmark Models

The standard GARCH model is given by:

$$r_t = \mu + a_t \tag{9}$$

$$a_t = \sigma_t \varepsilon_t \tag{10}$$

ISSN: 2692-7608 DOI: 10.6981/FEM.202311_4(11).0040

$$\sigma_t^2 = m + \alpha a_{t-1}^2 + \beta \sigma_{t-1}^2 \tag{11}$$

where r_t is the log return, μ is the conditional mean of return, σ_t^2 is the conditional variance of EUA futures returns.

The GARCH-MIDAS model is given by:

$$r_{i,t} = \mu + \sigma_{i,t}\varepsilon_{i,t} \tag{12}$$

$$g_{i,t} = (1-\alpha-\beta) + \alpha \frac{(r_{i-1,t}-\mu)^2}{\tau_t} + \beta g_{i-1,t} \tag{13}$$

$$\log(\tau_t) = m + \theta_1 \sum_{k=1}^{K} \psi_k(\omega_1)\log(RV_{t-k}) \tag{14}$$

The GARCH-MIDAS model is flexible and can be easily extended to incorporate RA. This can be done by incorporating RA into the long-term component process.

$$\log(\tau_t) = m + \theta_1 \sum_{k=1}^{K} \psi_k(\omega_1)\log(RV_{t-k}) + \theta_2 \sum_{k=1}^{K} \psi_k(\omega_2)\log(RA_{t-k}) \tag{15}$$

3.3. Volatility Forecasting Capability Assessments

As true volatility is unobservable, the evaluation and comparison of volatility forecasting models requires the use of proxies of true volatility. However, volatility proxies are often noisy (inaccurate) and are not perfect estimates of true volatility, which can lead to biased forecasting evaluations, with different loss functions yielding completely different optimal models. The loss funtion is commonly used to evaluate the forecasting accuracy of the competing models. We employ two popular loss functions, including mean squared error (MSE) and Quasi-likelihood (QLIKE), which can be defined as:

$$MSE = \frac{1}{T}\sum_{t=1}^{T}\left(RV_{t+1} - \hat{h}_{t+1}(m)\right)^2 \tag{16}$$

$$QLIKE = \frac{1}{T}\sum_{t=1}^{T}\left(\frac{RV_{t+1}}{\hat{h}_{t+1}(m)} - \log\left(\frac{RV_{t+1}}{\hat{h}_{t+1}(m)}\right) - 1\right) \tag{17}$$

where T is the size of prediction samples, RV_{t+1} and $\hat{h}_{t+1}(m)$ denote the measured volatility and forecasted volatility, respectively. and m stands for the GARCH, GARCH-MIDAS, GJR-GARCH-MIDAS and GJR-GARCH-MIDAS-RA models. It is worth noting that MSE and QLIKE are rubost loss functions that could provide a consistent ranking of the volatility models with a conditionally unbiased volatility proxy (Patton, 2011).

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4. Empirical Application

4.1. Descriptive Statistics

For the empirical investigation, we apply the daily (close-to-close) returns data of EUA futures and the monthly RA index data. The sampling stage of EUA futures data is selected from January 2, 2008 to March 31, 2021, with a total of 3410 trading days, and the data are obtained from Wind database of China. The time-varying risk aversion (RA) proxy used in this paper is the monthly RA index constructed by Bekaert et al. (2022) and sourced from the https://www.nancyxu.net/risk-aversion-index. The sampling period of the RA index data is from January 2008 to March 2021, matching the sample phase of the daily EUA futures data, for a total of 159 monthly data.

Table 1 reports the descriptive statistics of daily EUA futures returns ($r_{i,t}$) and monthly RA (RA_t) index. As can be seen from the table, the mean of $r_{i,t}$ is different from 0. Both series show the distributions of skewness are different from 0 (EUA futures returns series with skewness less than 0, that is, left-skewed; RA series with skewness greater than 0, that is, right-skewed) and excess kurtosis (kurtosis greater than 3). The Jarque-Bera statistics suggest that both sequences deviate from the normal distribution. The Ljung-Box Q statistics of EUA futures returns exhibits high persistence (or long-memory characteristic).

Table 1. Descriptive statistics

	Mean	Min	Max	Std.	Skew.	Kurt.	J-B	Q (10)
$r_{i,t}$	0.0002	-0.4347	0.2405	0.0315	-0.7665	18.3156	33652.0038	32.5790
RA_t	3.1798	2.4954	8.0302	0.9286	3.1474	13.9074	1050.6923	316.6165

Note: Std. is standard deviation, J-B is Jarque-Bera statistic, Q(10) is the Ljing-Box Q statistic with a lag of order 10.

Figure 1 is the time series plot of the daily return for EUA futures. It can be seen from the figure that the well-known behavior of volatility clustering (long-term memory characteristics) in EUA futures returns is apparent. In Figure 2, RA_t has tremendous fluctuations during the financial crisis in 2008 and the outbreak of COVID-19 in 2020, indicating that RA_t is sensitive to the changes in global financial market.

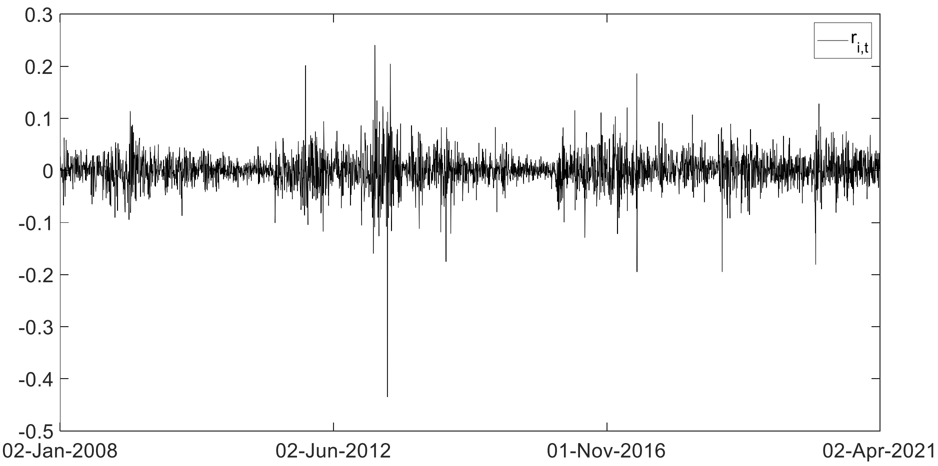

Figure 1. Time series plot of daily EUA futures returns $r_{i,t}$

ISSN: 2692-7608 DOI: 10.6981/FEM.202311_4(11).0040

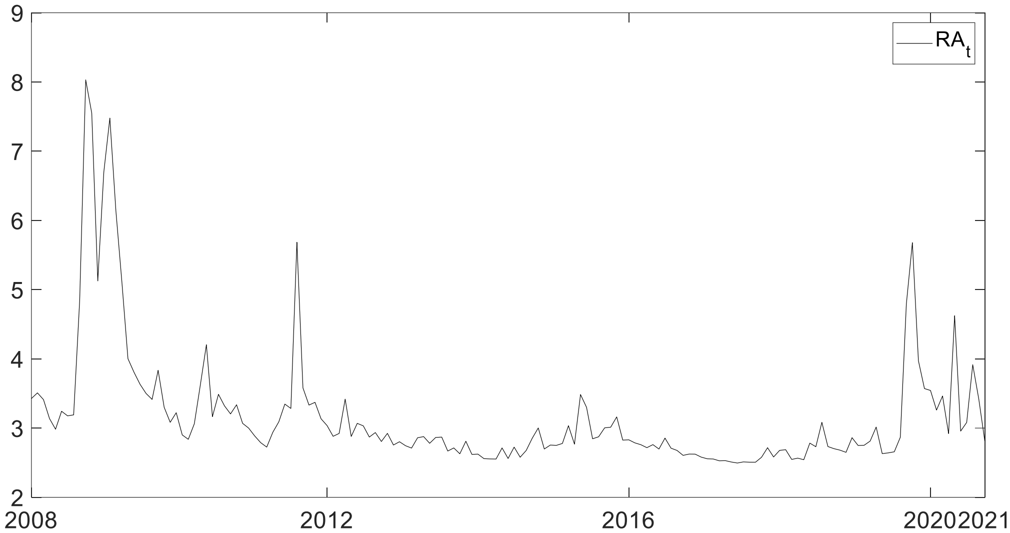

Figure 2. Time series plot of monthly RA_t index

4.2. Parameters Estimation

Table 2. Parameter estimation results

	GARCH	GARCH-MIDAS	GARCH-MIDAS-RA	GJR-GARCH-MIDAS	GJR-GARCH-MIDAS-RA
μ	0.0001 (0.0004)	0.0010 (0.0004)	0.0010 (0.0004)	0.0010 (0.0005)	0.0007 (0.0004)
m	0.0000 (0.0000)	-3.9721 (0.1186)	-3.0254 (0.0828)	-5.9192 (0.1333)	-3.4355 (0.0947)
θ_1		0.6319 (0.0262)	0.5703 (0.0188)	0.3727 (0.0158)	0.5605 (0.0207)
θ_2			-1.0377 (0.0260)		-0.8844 (0.0219)
ω_1		1.0043 (0.0169)	1.0098 (0.0185)	0.9494 (0.0103)	1.0078 (0.0165)
ω_2			6.0373 (0.3602)		6.7858 (0.3797)
α	0.1426 (0.0088)	0.1541 (0.0083)	0.1540 (0.0089)	0.0048 (0.0017)	0.1077 (0.0088)
β	0.8544 (0.0087)	0.8222 (0.0098)	0.8255 (0.0102)	0.9788 (0.0017)	0.8153 (0.0112)
γ				0.0183 (0.0023)	0.0952 (0.0133)
$Log-lik$	6246.4271	6254.5821	6255.9084	5950.7546	6265.3332
AIC	-12484.8542	-12497.1642	-12495.8168	-11887.5093	-12512.6665

Note: $Log-lik$ denotes log-likelihood value, AIC denotes Akaike information criterion. Standard errors (SE) of the maximum likelihood estimates are included in parentheses.

In order to estimate a model containing the MIDAS structure, the maximum lag order K of the MIDAS needs to be determined. Since the Beta weighting function used in this paper is flexible and the data will identify the optimal weights, as long as the chosen K is large enough, the estimation results to be robust with respect to the specific choice of the maximum number of

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lags included (Conrad and Loch, 2015). Given this, and also drawing on the work of Pan et al. (2017), we set $K=12$. That is, the lagging 12th order monthly low-frequency variable (historical 1-year information) is used to estimate the model containing MIDAS structure. Using the maximum likelihood estimation method, the parameters of GARCH, GARCH-MIDAS, GARCH-MIDAS-RA, GJR-GARCH-MIDAS, GJR-GARCH-MIDAS-RA models are estimated. The estimation results along with the standard error, log likelihood and Akaike information criterion (AIC) are presented in Table 2.

As can be seen from Table 2, the parameters estimation results of GARCH, GARCH-MIDAS, GARCH-MIDAS-RA, GJR-GARCH-MIDAS and GJR-GARCH-MIDAS-RA models, $\alpha+\beta$ ($\alpha+\beta+\gamma/2$), are estimated to be very close to 1, indicating that the volatility of EUA futures returns is highly persistent. In addition, the parameter γ is significantly positive, indicating that EUA futures returns have a reverse leverage effect (Wu et al., 2022). It is worth noting that the estimates of the coefficient θ_1 is significantly positive, indicating that the monthly realized volatility (RV) has a significant positive effect on the long-term volatility of EUA futures returns, that is, the monthly RV increases, the long-term volatility of EUA futures returns is expected to increase. In the two MIDAS models incorporated RA (GARCH-MIDAS-RA and GJR-GARCH-MIDAS-RA models), the estimates of the coefficient θ_2 is significantly negative, indicating that RA has a significant negative impact on the long-term volatility of EUA futures returns, that is, an increase in the level of RA predicts a decrease in the level of long-term volatility of EUA futures returns. One possible explanation is that when there is negative information appears in the EUA futures market, rational investors based on risk aversion preferences will purchase the safer financial asset to replace the EUA futures. The risk aversion of investors' behavior leads to a lower long-term volatility of EUA futures. According to the last two rows of Table 2, the GJR-GARCH-MIDAS-RA model corresponds to the largest likelihood function value and the smallest AIC, indicating that the GJR-GARCH-MIDAS-RA model has a better fit for EUA futures returns than other benchmark models.

5. VaR Evaluation

As good in-sample performance does not necessarily lead to good out-of-sample forecasts, we investigate the out-of-sample forecasting performance for practical purposes. This paper splits the sample into an in-sample estimation period (January 2, 2008 to December 31, 2018) and an out-of-sample forecast evaluation period (January 2, 2019 to March 31, 2021). For out-of-sample forecasting, we employ the rolling window procedure.

In Table 3, we present the results of backtesting, namely failure rate test and likelihood ratio tests of unconditional coverage (LR_{uc}) and conditional coverage (LR_{cc}) for VaR forecasts with probabilities α = 1%, 2.5% and 5%, using the GARCH, GARCH-MIDAS, GARCH-MIDAS-RA GJR-GARCH-MIDAS and GJR-GARCH-MIDAS-RA models with standard normal distribution. As we can see from Table 3, all models except the GJR-GARCH-MIDAS model predict VaR well at the 2.5% and 5% significance levels, the GARCH and GARCH-MIDAS models deteriorate obviously as the decrease of the probability α of VaR. Although the GARCH-MIDAS-RA model is rejected at the 5% significance level in the case of 1% VaR, the FR of GARCH-MIDAS-RA is less than that of GARCH and GARCH-MIDAS models at the most cases (the case of 1%, 2.5%, 5%), indicating that incorporating RA into model could decrease the failure rate of forecasting VaR for EUA futures. It is worth noting that the introduction of RA makes the GJR-GARCH-MIDAS model, which was rejected at three significant levels, all pass the test, and the failure rate of the GJR-GARCH-MIDAS-RA model is smaller than that of the GJR-GARCH-MIDAS model and closer to the

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corresponding significant level α, indicating that the model containing time-varying risk aversion can predict the VaR of EUA futures more accurately.

Table 3. Backtesting results

α	Model	FR	LR_{uc}	LR_{cc}
0.01	GARCH	0.0191	4.5245 (0.0334)**	4.5245 (0.1041)
	GARCH-MIDAS	0.0206	5.9186 (0.0150)**	5.9186 (0.0519)**
	GARCH-MIDAS-RA	0.0191	4.5245 (0.0334)**	4.5245 (0.1041)
	GJR-GARCH-MIDAS	0.0221	7.4587 (0.0063)***	7.4587 (0.0240)**
	GJR-GARCH-MIDAS-RA	0.0162	2.2202 (0.1362)	2.2202 (0.1362)
0.025	GARCH	0.0265	0.0623 (0.8029)	0.0623 (0.9693)
	GARCH-MIDAS	0.0280	0.2387 (0.6251)	0.2387 (0.8875)
	GARCH-MIDAS-RA	0.0250	0.0000 (0.9951)	0.0000 (1.0000)
	GJR-GARCH-MIDAS	0.0427	7.2319 (0.0072)***	11.6596 (0.0029)***
	GJR-GARCH-MIDAS-RA	0.0191	1.0372 (0.5953)	1.0372 (0.5953)
0.05	GARCH	0.0398	1.6058 (0.2051)	1.6058 (0.4480)
	GARCH-MIDAS	0.0412	1.1644 (0.2806)	1.1644 (0.5587)
	GARCH-MIDAS-RA	0.0398	1.6058 (0.2051)	1.6058 (0.4480)
	GJR-GARCH-MIDAS	0.0633	2.3503 (0.1253)	7.9730 (0.0186)**
	GJR-GARCH-MIDAS-RA	0.0383	2.1244 (0.1450)	2.1244 (0.3457)

Note: $1-\alpha$ is the confidence level, FR is the failure rate, LR_{cc} denotes the conditionally covered likelihood ratio statistic. * , ** and *** indicate that the model is rejected at the 10%, 5%, and 1% significance levels, respectively.

6. Conclusion

The REGARCH-MIDAS model can adequately portray the characteristics of financial market volatility, such as the asymmetry, long memory, and persistence of volatility exist in the financial markets. Meanwhile, empirical studies show that liquidity has an important impact on stock market volatility, and sufficient liquidity can help to maintain market stability and prevent financial systemic risk. In view of this, this paper proposes the REGARCH-MIDAS-RI model to model the volatility of the Chinese stock market under the framework of the REGARCH-MIDAS model by introducing stock market liquidity metrics RI. The model is flexible as it introduces RI through the MIDAS structure, which can fully capture the long memory feature of volatility as well as explain the expected variance of volatility to a certain extent.

Our empirical results suggest that RI has a significantly positive effect on the long-run volatility of Chinese stock market. Using four loss functions and MCS test, we show that the REGARCH-MIDAS model incorporating realized illiquidility (REGARCH-MIDAS-RI) provides more accurate out-of-sample forecasts of the Chinese stock market volatility compared to other competing models. Robustness checks suggest that the predictive power of the REGARCH-MIDAS-RI model is robust to alternative realized measure, alternative rolling windows as well as alternative MIDAS lags. Moreover, the economic value test shows that the REGARCH-MIDAS-RI model is the best measure of forecasting VaR at extreme probability levels for SSEC index in the Chinese stock market. Our findings highlight the importance of incorporating the realized illiquidility in forecasting the Chinese stock market volatility, which have significant implications for investors, policy makers and researchers.

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